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## Third Semester B.E. Degree Examination, Dec.2018/Jan.2019

### Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks: 80

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

#### Module-1

- 1 a. Define Tautology. Verify the following compound proposition is a tautology or not :  
 $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\sim r \rightarrow \sim (p \vee q)\}$ . (04 Marks)
- b. Check whether the following argument is valid or not :  
 If I study, then I will not fail in exam.  
 If I do not watch TV in the evenings, then I will study.  
 I failed in exam.  
 $\therefore$  I must have watched TV in the evenings. (04 Marks)
- c. Define : i) open sentence ii) quantifiers. Write the following proposition in symbolic form and find its negation : "All integers are rational numbers and some rational numbers are integers". (04 Marks)
- d. Give a direct proof of the statement, "For all integers K and l, if K and l are both even then  $K + l$  is even and  $Kl$  is even". (04 Marks)

#### OR

- 2 a. Define converse, inverse and contra positive of an implication. Hence find converse, inverse and contra positive for " $\forall x, (x > 3) \rightarrow (x^2 > 9)$ " where universal set is the set of real numbers R. (04 Marks)
- b. Using the laws of logic, prove the following logical equivalence :  
 $[(\sim p \vee \sim q) \wedge (F_0 \vee p) \wedge P] \Leftrightarrow p \wedge \sim q$ . (04 Marks)
- c. What are bound variables and free variables. Identify the same in each of the following expressions :  
 i)  $\forall y, \exists z \{ \cos(x + y) = \sin(z - x) \}$   
 ii)  $\exists x, \exists y \{ (x^2 - y^2) = z \}$ . (04 Marks)
- d. Verify the validity of the following argument : If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles. The triangle  $\triangle ABC$  does not have two equal angles.  $\therefore \triangle ABC$  does not have two equal sides. (04 Marks)

#### Module-2

- 3 a. Prove by mathematical induction  $1.3 + 2.4 + 3.5 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$ . (04 Marks)
- b. Give a recursive definition for each of the following integer sequence :  
 i)  $a_n = 7n$     ii)  $a_n = 2 - (-1)^n$  for  $n \in \mathbb{Z}^+$ . (04 Marks)
- c. How many positive integers can be formed by using the digits 3, 4, 4, 5, 5, 6, 7 to exceed 5,000,000? (04 Marks)
- d. In how many ways can we distribute seven apples and six oranges among four children so that each child receives at least one apple? (04 Marks)

OR

- 4 a. If  $F_0, F_1, F_2, \dots$  are Fibonacci numbers, then prove by induction  $\sum_{i=1}^n \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$ . (04 Marks)
- b. A sequence  $\{a_n\}$  is defined recursively as  $a_1 = 7$  and  $a_n = 2a_{n-1} + 1$  for  $n \geq 2$ . Find  $a_n$  in explicit form. (04 Marks)
- c. Find the number of arrangements of all the letters in the word "TALLAHASSEE". How many of these arrangements have no adjacent A's? (04 Marks)
- d. Find the coefficient of  $w^3 x^2 y z^2$  in the expansion of  $(2w - x + 3y - 2z)^8$ . (04 Marks)

**Module-3**

- 5 a. Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that  $A \times (B - C) = (A \times B) - (A \times C)$ . (04 Marks)
- b. Let f and g be two functions from R to R defined by  $f(x) = 2x + 5$  and  $g(x) = \frac{x-5}{2}$ . Show that f and g are invertible to each other. (04 Marks)
- c. Define partition of a set. If R is a relation defined on  $A = \{1, 2, 3, 4\}$  by  $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$ , determine the partition induced by R. (04 Marks)
- d. Let  $A = \{a, b, c\}$ ,  $B = P(A)$  where P(A) is the power set of A. Let R be a subset relation on A. Show that  $(B, R)$  is a POSET and draw its Hasse diagram. (04 Marks)

OR

- 6 a. Let R be an equivalence relation on set A and  $a, b \in A$ . Then prove the following are equivalent :  
 i)  $a \in [a]$   
 ii)  $a R b$  iff  $[a] = [b]$   
 iii) if  $[a] \cap [b] \neq \phi$  then  $[a] = [b]$ . (04 Marks)
- b. Prove that a function  $f: A \rightarrow B$  is invertible iff it is one – one and onto. (04 Marks)
- c. State Pigeonhole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13. (04 Marks)
- d. Show that the set of positive divisors of 36 is a POSET and draw its Hasse diagram. Hence find its i) least element ii) greatest element. (04 Marks)

**Module-4**

- 7 a. Out of 30 students in a hostel, 15 study history, 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. (04 Marks)
- b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all these derangements. (04 Marks)
- c. Find the rook polynomial for the following board [refer Fig.Q7(c)] :

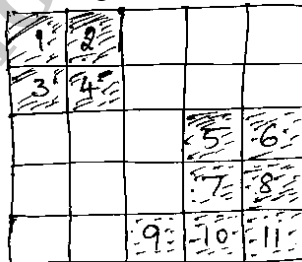


Fig. Q7(c)

(04 Marks)

- d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. (04 Marks)

OR

- 8 a. Determine the number of integers between 1 and 300 (inclusive) which are  
 i) divisible by exactly two of 5, 6, 8    ii) divisible by at least two of 5, 6, 8. (04 Marks)
- b. In how many ways can the integers 1, 2, - - - , 10 be arranged in a line so that no even integer is in its natural place. (04 Marks)
- c. An apple, a banana, a mango and an orange are to be distributed to four boys  $B_1, B_2, B_3, B_4$ . The boys  $B_1$  and  $B_2$  do not wish to have apple, the boy  $B_3$  does not want banana or mango,  $B_4$  refuses orange. In how many ways the distribution can be made so that no boy is displeased? (04 Marks)
- d. Solve the recurrence relation  $F_{n+2} = F_{n+1} + F_n$  for  $n \geq 0$  given that  $F_0 = 0, F_1 = 1$ . (04 Marks)

**Module-5**

- 9 a. Define the following with an example for each :  
 i) Complete graph    ii) regular graph    iii) bipartite graph    iv) complete bipartite graph. (04 Marks)
- b. Define isomorphism of two graphs. Verify the following graphs are isomorphic or not :  
 [Refer Fig.Q9(b)] (04 Marks)

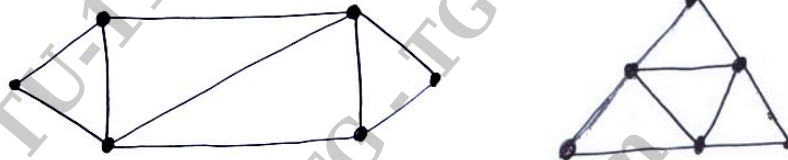


Fig.Q9(b)

- c. Show that a tree with  $n$  vertices has  $n - 1$  edges. (04 Marks)
- d. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. (04 Marks)

OR

- 10 a. Explain Konigsberg bridge problem. (04 Marks)
- b. Define the following with an example :  
 i) subgraph    ii) spanning subgraph  
 iii) induced subgraph    iv) edge-disjoint and vertex-disjoint subgraphs. (04 Marks)
- c. If a tree  $T$  has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in  $T$ . (04 Marks)
- d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. (04 Marks)

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